**Lecture Note-04**

**Integration using Cauchy’s Residue** **Theorem (CRT)**

Two main reasons account for the importance of integration in the complex plane. The practical reason is that complex integration can evaluate certain real integrals appearing in applications that are not accessible by real integral calculus. The theoretical reason is that some basic properties of analytic functions are difficult to prove by other methods. Complex integration also plays an important role in connections with special function, such as the gamma function, the error function, various polynomials and others, and the application of these functions in physics.

**Cauchy’s Integral Formula:**

If a function  is analytic within and on a simple closed contour  and if  is any point interior to then,

**- - - - - - - - - (1)**

**Special case :** If is not an interior point of the contour  then **.**

Differentiating n-1 times w.r.to

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**Definition of singular point (of an analytic function):**

A point at which an analytic function is not defined, i.e., at which fails to exist, called a singular point or pole or singularity of the function.

**Example 7.1:** If then , 3 are the singular points of 

**Residue Finding Method:**

If is analytic inside and on a simple closed curve *C* except at pole or has singularity at  of order 1, then

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If is analytic inside and on a simple closed curve *C* except at pole or has singularity at of order ***m***, then



**Cauchy Residue Theorem:**

If is analytic inside and on a simple closed curve *C* except at a finite number of singular points  inside *C*, then

|  |  |  |
| --- | --- | --- |
| **Example 7.2:** Evaluate by CRT , where C is the circle  **Solution**: For singular point,    Singular point is a pole of order 2. The point lies inside the circle .  Residue at the point  is,          So by CRT we know,    **Example 7.3:** Evaluate the contour integral by CRT, where C is the circle   |  |  | | --- | --- | | **Solution:** The poles or singularities of  are as follows:  A pole of order 3 at . This pole lies inside the contour C.  Residue at the point of order 3 is given by        So by CRT we know, =0. |  |   **Sample Exercise Set on Cauchy residue theorem: 4.1**     1. (i) Find all the singular points of the following functions , and show the points in the argand diagram, where =, .   (ii) Find all the singular points of the following functions , *f(z)* and show them in the argand diagram,then find corresponding residues : = , .   1. State Cauchy’s integral formula and Cauchy’s residue theorem (CRT). For each of the followings sketch the indicated path and hence evaluate applying Cauchy’s residue theorem (CRT) and Cauchy’s Integral Formula(CIF), ( if possible):   (a) , is the circle .  (b) , consists of .  (c) , where is the circle  3. Evaluate the followings applying Cauchy’s residue theorem (CRT) (if possible):  (a) Evaluate the integrals along the contour as given in the figures:  (i) (Fig. 1), (ii)(Fig. 2), (iii) (Fig. 3).  C:\Users\Miraz\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.Word\Untitled.jpg  Fig.1 Fig.2 Fig.3  4. For the followings sketch the indicated path and hence evaluate applying Cauchy’s residue theorem (CRT) if possible:  (a), is the contour as (i) , (ii).  (b) where is the circle  (c) where is the circle .  (d) where is the circle .  Reference Book: Advanced Engineering Mathematics (10th edition) by Erwin Kreyszig, Herbert Kreyszig, Edward J. Norminton, published by John Wiley & Sons, Inc |